Big Things that Squeeze Round Corners

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No man is an island, entire of itself. But almost every man is a furniture-mover from time to time. And woman too of course.

As furniture movers, we always ask ourselves: ‘will that bed/desk/wardrobe go up that staircase?’ After grunting and shoving for a few minutes, we wonder ‘hmm, would it go round the corner if I sawed that leg off?’ After sawing off all four legs and hacking away the banisters with an axe, we give up and decide to move into a different house.

In this article, I explore the question, ‘what is the largest piece of furniture that can be squeezed round a corner?’

Even in a two-dimensional flatland, I don’t know a mathematical technique for tackling the problem. The task is to make a rigid object with the largest possible area that can be taken round a corner in a flatland corridor of width 1.

The figure shows a corridor of width 1, and two simple objects, a circle and a square of areas $\pi/4 \simeq 0.79$ and 1, respectively, both of which can slide along the corridor and go round the corners.

The third figure shows a semicircle of radius 1 and area $\pi/2 \simeq 1.57$. 
The second figure shows a telephone-shaped object, whose area is $\pi/2 + 2/\pi \approx 2.21$. It is built from two quarter-circles of radius 1 and a rectangle of size $1 \times (2/\pi)$, with a semi-circle chomped out of it. This is the largest object I have found that can go round the corner.

Is it the largest? Perhaps this field of furniture design is still an active research area.

Reading the literature...

(Added Thu 4/9/03.)

I usually read the literature a few years after I started working on a problem. The telephone-shaped object above is called the Hammersley sofa, and it’s not the largest possible piece of furniture. The ‘Gerver sofa’ looks like the Hammersley sofa, but has a more complicated and smoother perimeter. Whereas the Hammersley sofa has area 2.2074 square units, the area of the Gerver sofa is 2.2195, an increase of 0.5%. It remains an open problem to prove whether the Gerver sofa is the largest shape that will fit through the corridor. Internet references: http://www.studyworksonline.com/cda/content/article/0,,EXP1755_NAV2-95_SAR1766,00.shtml http://www.mathcad.com/library/constants/sofa.htm

Another puzzle

Here’s another puzzle relating to furniture-moving.

The league for the confounding of furniture-movers devotes its energy to making pieces of two-dimensional furniture that cannot be moved down unit-width corridors – not even down straight ones. And they are interested in minimizing the amount of materials that goes into their objects.

For example, an L-shape (right-angled) piece of stiff wire of length slightly more than $2\sqrt{2}$ cannot be moved down the corridor. How much better than this can you do? If you are allowed to weld pieces of stiff wire to each other, I think the optimal solution is easy to find. But purists in the league like to make their unmoveable objects out of a single bent piece of wire, with the shortest possible length.

References for the Moving Sofa constant


I. Stewart, Another Fine Math You’ve Got Me Into..., W. H. Freeman, 1992; MR 93i:00003.